MORE PARKING FOR LESS

Preliminary round assignment 22st Mathematics A-lympiad

November 19, 2010

**Instructions for the preliminary round assignment for
the Mathematics A-lympiad 2010/2011**

This Mathematics A-lympiad assignment consists of two introductory assignments, and three final assignments.

General advice for working on this assignment
- First read the entire text of the assignment so that you'll know everything you have to do.
- **Keep an eye on the time you use for the introductory and follow-up assignments. Make sure you have enough time left for the final assignment.** Divide tasks where possible and confer with each other when needed.
- If you have divided your tasks, discuss the results of the introductory assignments with each other before you start on the final assignment.
- It is important for the first two of the final assignments that you first set up a work schedule for the research you will do. Every assignment states explicitly what you have to hand in.

The answers to the introductory assignments (1 and 2) should not be included in the elaboration of the final assignments (3, 4 and 5). Add the results of the introductory assignments as an appendix.

Hand in:
- Check the descriptions for the final assignments
- As appendices: the results of the introductory assignments (assignment 1 and 2)

The jury will be given copies of your work. Of course these copies must be legible. Therefore write and draw with a black pen, only print on A4 paper, and do not use pencils for any of your drawings. If in doubt, make a test copy!

Assessment:
The jury will focus especially on:
- legibility and clarity of the final assignment,
- whether the work is complete,
- the use of mathematics,
- the arguments used and justification of choices (realism may play a part here),
- the level to which the assignment has been done,
- presentation: form, legibility, structure, use and function of appendices, etc.

Have fun and good luck!

More parking for less

Schiphol Airport uses different price rates for its parking: not everyone pays the same price per day. Below you can see an example of an advert to illustrate this. In this Alympiad assignment you will investigate how to make the yield of a 500-place car park (for instance at an airport) as high as possible.

Schiphol Smart Parking

Zoekt u een veilige, goedkope parkeerplek op de luchthaven? Dan is Schiphol Smart Parking de voordeligste manier om uw auto te parkeren op Schiphol. Hiervoor moet u minimaal 1 week voor vertrek online reserveren en betalen met uw credit card.

Rekenvoorbeeld Schiphol Smart Parking

	Normaal tarief	Schiphol Smart Parking tarief
3 dagen	€ 52,50	€ 37,50*
8 dagen	€ 85,00	€ 49,50
15 dagen	€ 130,50	€ 89,50
22 dagen	€ 176,00	€ 110,00

* Dit tarief is te reserveren tot 2 weken voor vertrek

Check uw tarief, de beschikbaarheid en reserveer

In heel 2010 krijgt u nog meer korting op Smart Parking wanneer u vroeg boekt.

Boekt u bijv. min. 6 weken van te voren, betaalt u voor 22 dagen maar EUR 95,– (4,32 per dag).

Het aanbod van deze extra vroegboekkorting is beperkt. Wees er dus snel bij want op=op!

Reserveer nu!

EXTRA vroegboek korting!

Introduction

One way to easily manage a 500-place car park could be the following:
Set a fixed daily rate, for example € 10,00, and wait to see how many cars come in every day and for how long they stay. Once all spaces are taken, new arrivals are out of luck, while they might have been willing to pay more than € 10,00 per day.

That is not the optimal approach for a high yield. There will be some days when many places remain empty. In addition, the rate may be too high for people who want to park for longer because they are going on holiday. For them it is valid to say that the lower the rate, the more will come. On the other hand there will be business people who don't mind the higher rate, as long as they can be certain that they will have a space. These business users will normally only stay for a few days, and it would be a shame if you had to disappoint these high-paying customers because you have already sold a parking space for a lower rate.
In short:
- you must reserve a number of spaces against a higher rate, for the users for whom you are fairly certain that they will come as long as they know they have a space (regardless of the price), and:
- you must be able to set a lower rate for holiday-goers who can reserve a space in advance.

As long as you have enough data, you can determine this again for every day. In this Alympiad assignment you will investigate how best to fill a car park by varying the rates.

Part 1: The first week

We assume two kinds of users for our car park:

- business users who pay a high set rate, and stay for a short time
- holiday users who are drawn in by the lower rates (discounts) and stay long

Research shows a pattern for the days on which both types of users arrive, see the diagram below. This diagram is for a normal week, outside holiday periods. On average in such a week there are 500 business users who on average leave their cars for two days, and 200 holiday users who on average leave their cars for four days. For now we are assuming the following:

- anyone who parks in the car park will do so in the morning;
- anyone who picks up their car will do so in the evening.

diagram 1
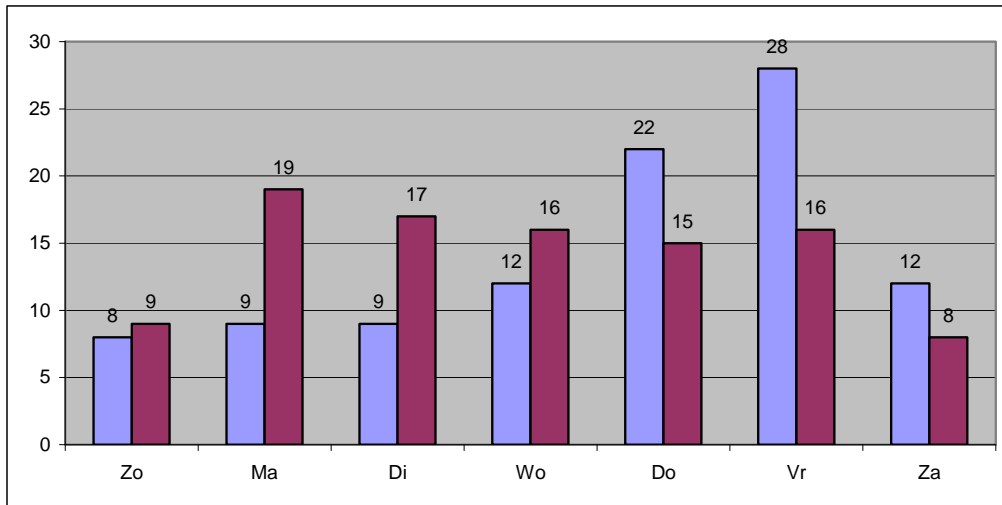
█ (light) percentage of holiday users arriving on a week day
(200 holiday users per week; cars are there for an average of 4 days)

█ (dark) percentage business users arriving on a week day
(500 business users per week; cars are there for an average of 2 days)

One thing you can see in the diagram is that 15% of business users arrive on a Thursday (and they pick up their car again on Friday night).

Introductory assignment 1
We start on Sunday morning with an empty car park.
How many cars from business users can we expect to be in the car park on Wednesday afternoon in total? And on Saturday night?

Introductory assignment 2
It's expected that the car park won't be empty on Sunday morning the week after. For instance there will be cars from holiday users who arrived on Friday. After a certain day, the situation will become stable and the numbers of users will be the same for every Monday, every Tuesday and so on.

Calculate (for the stable situation given above) the expected weekly yield for the car park with a set rate of € 10,00 per day for business users and € 6,00 for holiday users.

Hand in: an appendix with the elaboration of both introductory assignments

Part 2: the rate for holiday users during a holiday period

We are still assuming 500 business users per week, divided over the days based on the earlier diagram, and paying € 10,00 per day. Part of the car park is reserved for these business users. The remaining places can be taken up by holiday users.

In the previous we assumed 200 holiday users per week. In the weeks with school holidays we assume that the demand for parking won't be limited to 200 places for holiday users. The number of holiday users will depend on the rate.

How many holiday users are expected to come and what rate are they willing to pay to park? Most holiday-goers won't automatically take their car to the airport. They will have someone drop them off, or take the train if they think the price to park is too high. These holiday-goers can be tempted with low rates: you can find an example in the advert on page 3 of this assignment.

A study has been made into the number of holiday users that can be expected in a week during the holiday season at a given rate. This study resulted in the following diagram:

diagram 2

It should be obvious, given the size of the car park, 500 places, that a 1 euro rate for holiday users isn't a good idea... but what is? De € 6,00 from the first two assignments? Do you

change it or not? Also, it could be that you might get an even higher expected weekly yield if you disappoint some business users, or that it's to your advantage for the expected weekly yield to keep reserved places for all business. The percentages in diagram 1 are still valid.

Final assignment 3
a
Think in advance of an approach for researching what rate for the holiday users will lead to the highest yield for the car park. Give a detailed description of your approach.
b
Do your research according to the approach you came up with, check whether this gives the desired results, and if necessary adapt your approach.

Hand in: a detailed description of the approach for your research, illustrated with (preliminary) calculations and a critical review of the quality of the outcome of your research.

Part 3: the business user

Up to now we assumed 500 business users per week. But do we really have 500 business users? You can never know in advance... It's important to be able to say something about it: after all, holiday users book several weeks in advance, while business users just turn up. So you need to reserve spaces for business users. But how many? Empty places don't yield any profit!
Research shows that there is a distribution of probability for the number of business users. It shows that you can almost certainly count on having those 500 business users every week , but that there will be more in some weeks, and very rarely there are close to 700!
The research resulted in the following diagram:

diagram3
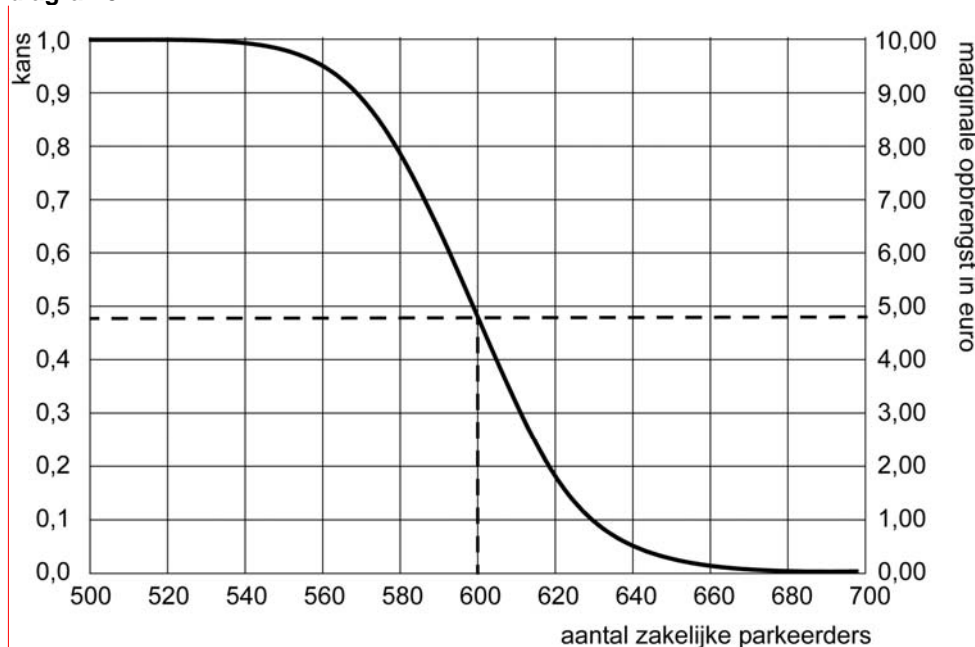
aantal zakelijke parkeerders

In this diagram you can see what the expected result is of reserving extra parking spaces for business users. The more parking spaces have already been reserved, the lower the

expected extra (business) yield is for a parking space. We will show that using the following example:

In the diagram you can see that the chance of having at least 600 business users is 0,48. The expected extra business yield of reserving 600 (instead of 599) parking spaces at a rate of €10,00 x 0,48 = € 4,80.

So there are three factors that play a part in determining the optimum yield:
- the distribution of arrivals over the week (diagram 1)
- the number of holiday users depending on the rate (diagram 2)
- the distribution of probability of the number of business users (diagram 3)

Final assignment 4
Research the effect of the given distribution of probability in diagram 3 on the optimal situation that you found in final assignment 3. So now you're no longer getting yields, but expectations of yield. (That is to say: a result multiplied with the chance of that yield) Again, first come up with an approach, then describe your approach, and perform your research according to that approach.

Hand in: an elaborate description of the research, illustrated with (preliminary) calculations and a critical review of the quality of the results of your research.

Final assignment 5
In the previous assignments you gained an impression of a method of making as much money as possible from a car park, where you're dealing with people who book in advance, and want to pay as little as possible (the holiday users) and people who almost don't mind what they pay – as long as they have a parking space (the business user).
Write a qualitative analysis of this method: where are the weaknesses, which data are easy or hard to determine, what data would you like to have, etc. Include your own experiences in this analysis and refer back to the other final assignments to support your conclusions.

Hand in: a critical review of this method in the form of a short article in a professional journal. This article will consist of illustrative calculations for no more than half the length.